**A Higher-Order Factor Analysis of the Short Form of the Attitude Toward Mathematics Inventory Using Elementary Student Data**

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In 2014, a capstone project examined if a 12-week treatment that focused on instruction and character traits would positively affect students' feelings toward school and mathematics and their academic performance in mathematics. In addition, an attempt was made to understand if an increase in perseverance led to a change in attitude toward mathematics, perceptions about themselves as students, and standardized test scores.

One of the instruments used in that capstone was the Attitudes Toward Mathematics Inventory (ATMI; Tapia, 1996). However, there were two limitations to the inventory's use during this capstone project. First, the ATMI was designed and tested for older students, specifically middle and high school students. Second, the data were evaluated surface-level and not with any substantial qualitative measures to check for validity.

In 2012, Lim and Champan aimed to shorten the ATMI inventory as they felt it was too long at 40 questions. They ran an analysis on the inventory to determine if the test to get under a 10-minute administration level. They were successful, scaling the inventory down to 19 questions. However, this was not tested on elementary students.

An analysis was conducted using the capstone project data to shorten the inventory further for use with grades 3-5 students. It revealed that the subfactor "self-confidence" needed to be removed, thus bringing it to 14-items.

The study analysis seeks to examine the possibility of shortening the inventory to under ten questions while maintaining validity.

## Study Analysis History

As Lim and Champan sought to eliminate items, they determined after research and analysis that the inventory could be trimmed to 19 items. The following are descriptions of the SF-ATMI (Lim & Champan, 2012) survey items sorted by the subscales identified in the rotated component matrix.

Enjoyment of Mathematics:

- ENJ2 I have usually enjoyed studying mathematics in school
- ENJ4 I like to solve new problems in mathematics
- ENJ6 I really like mathematics
- ENJ7 I am happier in a mathematics class than in any other class
- ENJ8 Mathematics is a very interesting subject

Motivation to do Mathematics:

- MOT1 I am confident that I could learn advanced mathematics
- MOT3 I am willing to take more than the required amount of mathematics
- MOT4 I plan to take as much mathematics as I can during my education
- MOT5 I believe studying math helps me with problem-solving in other areas

Perceived Value of Mathematics

- VAL1 Mathematics is a very worthwhile and necessary subject
- VAL10 A strong math background could help me in my professional life
- VAL4 Mathematics is important in everyday life
- VAL5 Mathematics is one of the most important subjects to study
- VAL6 High school mathematics courses would be very helpful no matter what I decide to study

Self-Confidence of Mathematics

- SC10 I am always confused in my mathematics class
- SC13 I feel a sense of insecurity when attempting mathematics
- SC3 Studying mathematics makes me feel nervous
- SC5 I am always under a terrible strain in a math class
- SC7 It makes me nervous to even think about having to do a mathematics problem

During the previous analysis, removal of the self-confidence factor significantly increased the survey's Cronbach's Alpha from .694 to .891, a jump from minimally acceptable to very good, according to DeVellis (1991) guidelines for acceptable internal consistency reliabilities. Therefore, a decision was made to remove the self-confidence factor. The Self-Confidence of Mathematics subscale negativity corrected item-total correlated to the survey as a whole and the other factors individually. Removal of that subscale allowed the reset of the data analysis to continue, as seen in Figure 1.

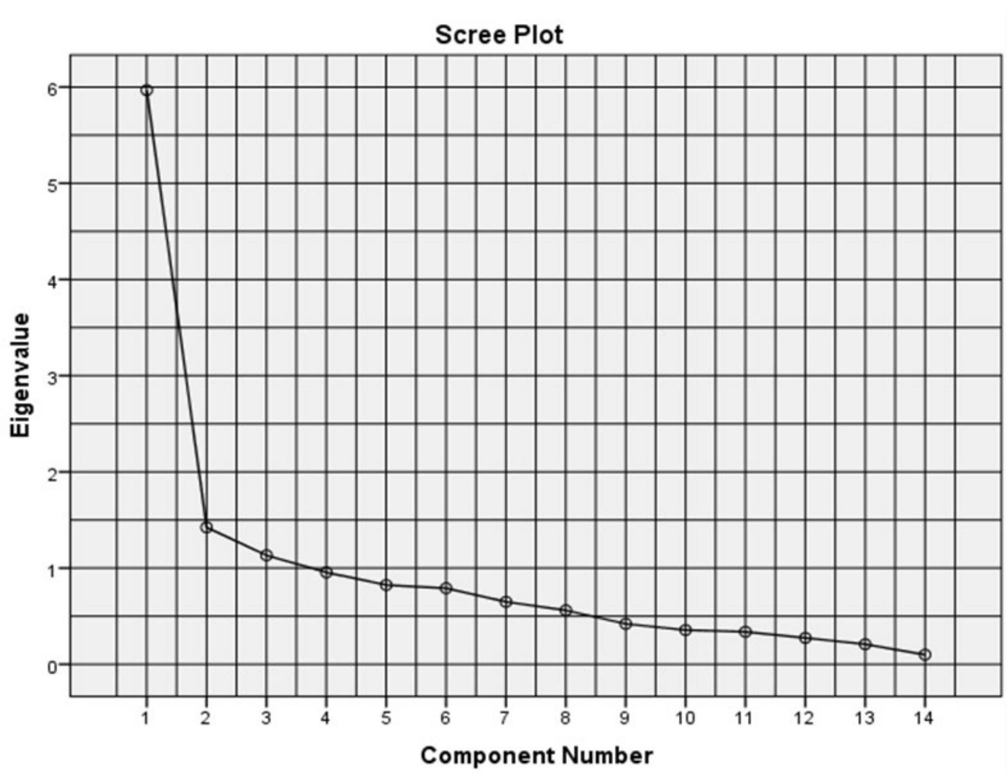


*Figure 1.* Scree Plot for a 14-item SF-ATMI, Self-Confidence Factor Removed

The first analysis of the SF-ATMI produced the following subscales:

| Question | Item Code | Item Description | Original SF-ATMI Code |
| --- | --- | --- | --- |
| 1 | VAL1 | Mathematics is a very worthwhile and necessary subject | Perceived Value of Mathematics |
| 24 | ENJ2 | I have usually enjoyed studying mathematics in school | Enjoyment of Mathematics |
| 29 | ENJ6 | I really like mathematics | Enjoyment of Mathematics |
| 30 | ENJ7 | I am happier in a mathematics class than in any other class | Enjoyment of Mathematics |
| 31 | ENJ8 | Mathematics is a very interesting subject | Enjoyment of Mathematics |
| | | ------------------------------------------------------------------- | |
| 26 | ENJ4 | I like to solve new problems in mathematics | Enjoyment of Mathematics |
| 23 | MOT1 | I am confident that I could learn advanced mathematics | Motivation to do Mathematics |
| 32 | MOT3 | I am willing to take more than the required amount of mathematics | Motivation to do Mathematics |
| 33 | MOT4 | I plan to take as much mathematics as I can during my education | Motivation to do Mathematics |
| 36 | MOT5 | I believe studying math helps me with problem-solving in other areas | Motivation to do Mathematics |
| 5 | VAL4 | Mathematics is important in everyday life | Perceived Value of Mathematics |
| 6 | VAL5 | Mathematics is one of the most important subjects to study | Perceived Value of Mathematics |
| 7 | VAL6 | High school mathematics courses would be very helpful no matter what I decide to study | Perceived Value of Mathematics |
| 39 | VAL10 | A strong math background could help me in my professional life | Perceived Value of Mathematics |

## Factor Analysis

A factor analysis was performed on the 14 survey items to begin the data analysis. First, these items were tested using the demographic race. Next, a rotated component matrix using varimax was composed to determine the number of constructs. Two subscales emerged and were noted for further analysis (see Table 1). Further analysis is needed to examine these two new factors and examine any relationships within them.

Table 1

*Rotated Component Matrix Displaying Two Constructs*

| | 1 | 2 |
|---|---|---|
| MOT4 | .811 | .040 |
| VAL10 | .741 | .125 |
| MOT1 | .694 | .340 |
| MOT3 | .637 | .390 |
| MOT5 | .604 | .315 |
| ENJ4 | .555 | .200 |
| VAL4 | .503 | .279 |
| VAL6 | .475 | .440 |
| ENJ6 | .100 | .885 |
| ENJ8 | .197 | .865 |
| ENJ2 | .186 | .629 |
| VAL1 | .420 | .590 |
| ENJ7 | .282 | .567 |
| VAL5 | .467 | .552 |

Extraction Method: Principal Component Analysis.
Rotation Method: Varimax with Kaiser Normalization.
a. Rotation converged in 3 iterations.

The initial findings of the rotated component matrix show that two subscales are formed from the three primary factors used: Enjoyment of Mathematics, Motivation to do Mathematics, and Perceived Value of Mathematics.

**Subscale 1**

- Q33 MOT4 I plan to take as much mathematics as I can during my education
- Q39 VAL10 A strong math background could help me in my professional life
- Q23 MOT1 I am confident that I could learn advanced mathematics
- Q32 MOT3 I am willing to take more than the required amount of mathematics
- Q36 MOT5 I believe studying math helps me with problem-solving in other areas
- Q26 ENJ4 I like to solve new problems in mathematics
- Q5 VAL4 Mathematics is important in everyday life
- Q7 VAL6 High school mathematics courses would be very helpful no matter what I decide to study

**Subscale 2**

- Q29 ENJ6 I really like mathematics
- Q31 ENJ8 Mathematics is a very interesting subject
- Q24 ENJ2 I have usually enjoyed studying mathematics in school
- Q1 VAL1 Mathematics is a very worthwhile and necessary subject
- Q30 ENJ7 I am happier in a mathematics class than in any other class
- Q6 VAL5 Mathematics is one of the most important subjects to study

## Cluster Analysis

The data were reanalyzed to check for validity. This analysis was performed to ensure that the data yielded two factors; more than two factors would have prompted a need for higher-order factor analysis. Once it was determined that a higher-order analysis was not necessary, further analysis was conducted using cluster analysis, multidimensional scaling, and cluster quality.

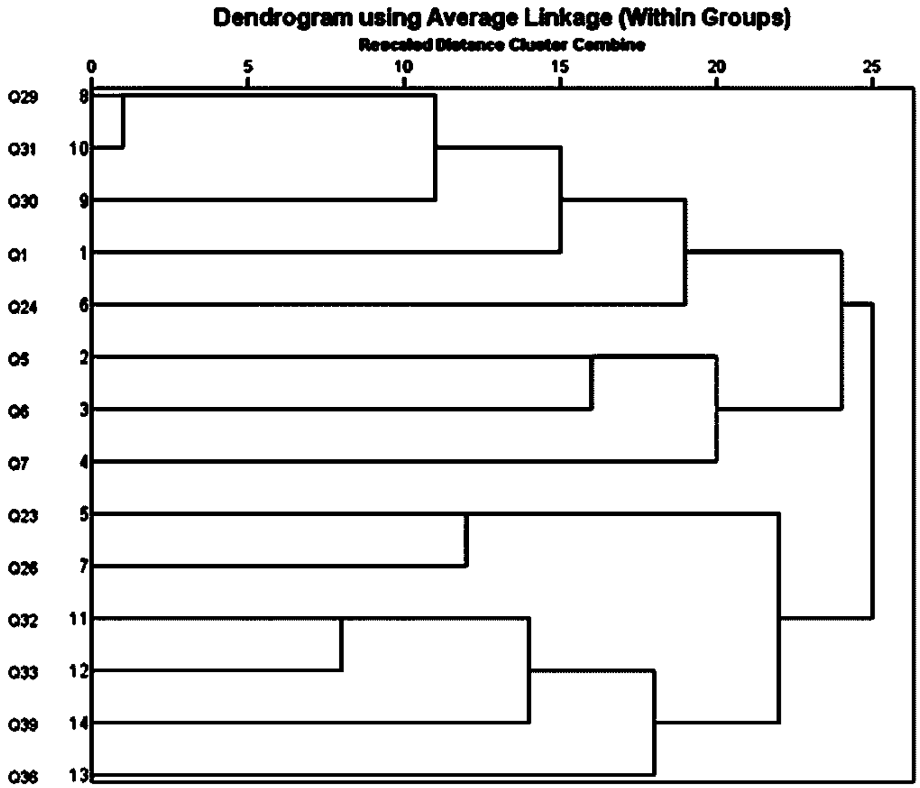


*Figure 2.* Dendrogram of Survey Items.

The Dendrogram (see Figure 2) reveals a two-cluster structure. Visually, it appears the two clusters are separated between Q7 and Q23. This separation is inconsistent with the structure of the two clusters. The clusters aligned with the two factors identified in the factor analysis show the division between Q5 and Q24.

In addition to appearing in the component one branch, Q5-7 are rearranged from the rotated component matrix - "Mathematics is important in everyday life" (Q5, VAL4); high school mathematics courses would be very helpful no matter what I decide to study" (Q7, VAL6), and "Mathematics is one of the most important subjects to study" (Q6, VAL5). Further clarification was sought, and analysis was conducted to understand the survey item analysis discrepancy.

## Multidimensional Scaling

To begin the multidimensional analysis, a one-dimensional analysis using ALSCAL on the 14-items. The one-dimensional analysis shows items Q5, Q26, Q23, Q29, Q24, and Q30 further away from the other items of approximate distance negatively and positively from zero. (see Table 2 and Figure 2.)

Table 2

| Dimension 1 | | | |
|---|---|---|---|
| VAL1 | -0.14 | ENJ6 | 0.72 |
| VAL4 | -1.029 | ENJ7 | 1.195 |
| VAL5 | -0.103 | ENJ8 | 0.426 |
| VAL6 | -0.27 | MOT3 | 0.291 |
| MOT1 | -0.691 | MOT4 | 0.044 |
| ENJ2 | 700 | MOT5 | -0.299 |
| ENJ4 | -0.94 | VAL10 | 0.096 |

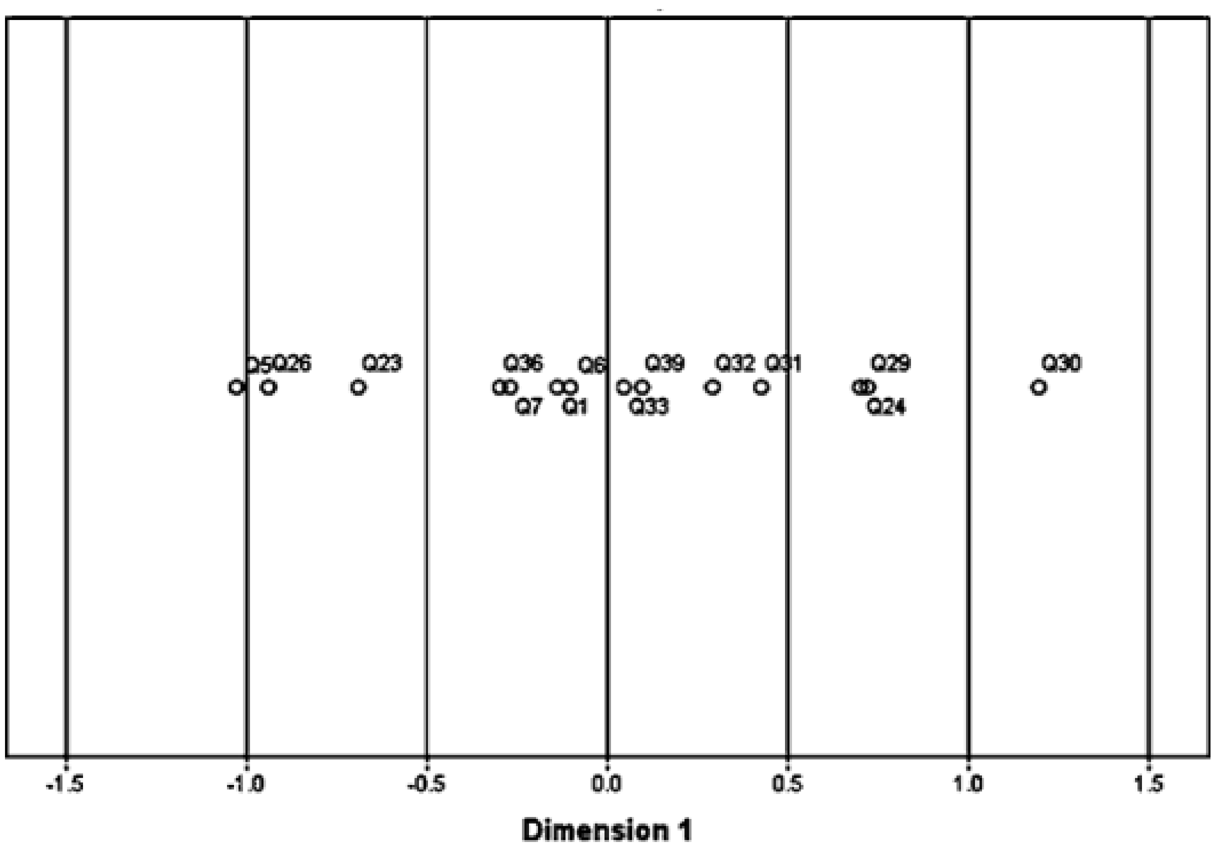


*Figure 2*. Survey Items Represented on a One-Dimension Plane.

The data was further examined using an ALSCAL two-dimensional analysis (see Figure 3 and Table 3). The two dimensions yielded an RSQ was .81999, meaning that any dimension would be represented. Therefore, the stress levels or measure of goodness of fit of the mapping solution to the data is .18975, a fair goodness-of-fit result.

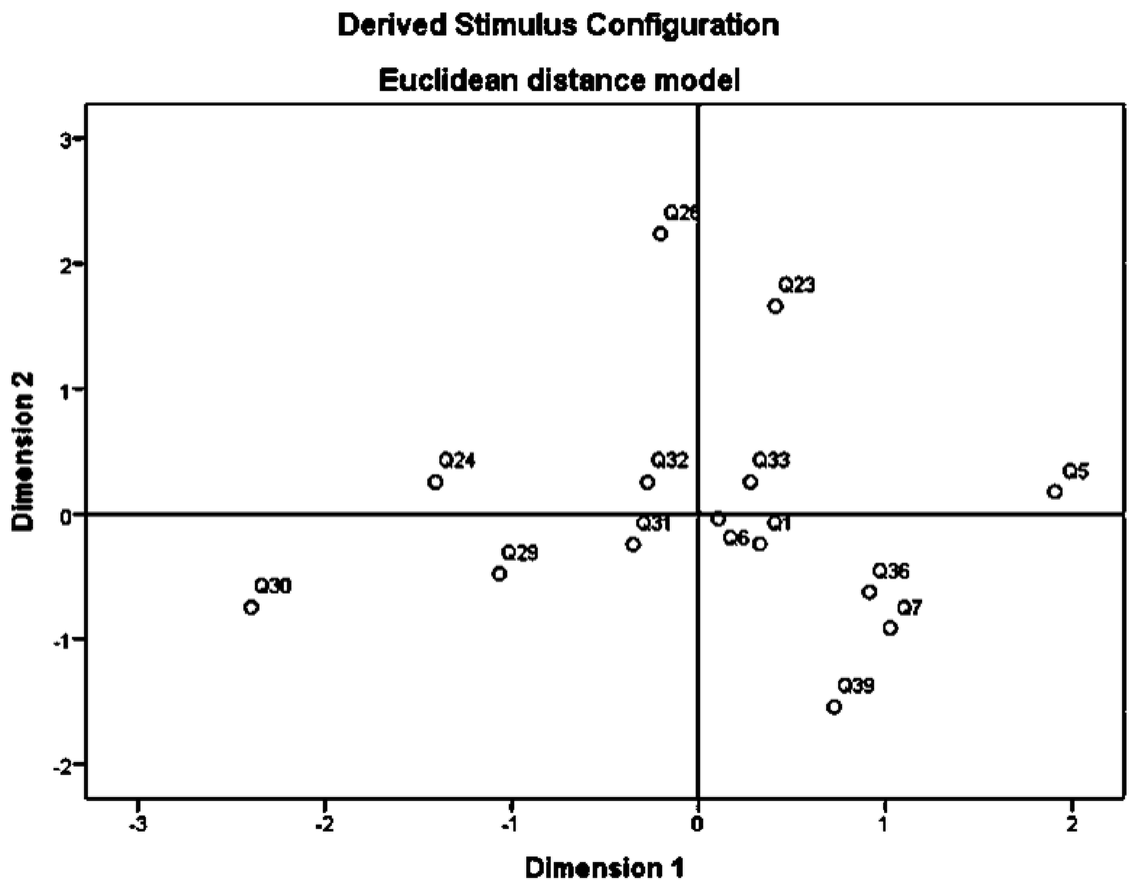


*Figure 3*. Survey Items Represented on a Two-Dimension Plane.

Table 3
*Two-Dimension Analysis Results with Stimulus Coordinate Values*

| Stimulus Number | Stimulus Name | 1 | 2 |
|---|---|---|---|
| 1 | Q1 | .3283 | -.2413 |
| 2 | Q5 | 1.9067 | .1773 |
| 3 | Q6 | .1059 | -.0402 |
| 4 | Q7 | 1.0242 | -.9113 |
| 5 | Q23 | .4116 | 1.6608 |
| 6 | Q24 | -1.4072 | .2541 |
| 7 | Q26 | -.2041 | 2.2375 |
| 8 | Q29 | -1.0657 | -.4799 |
| 9 | Q30 | -2.3937 | -.7486 |
| 10 | Q31 | -.3494 | -.2449 |
| 11 | Q32 | -.2737 | .2509 |
| 12 | Q33 | .2769 | .2548 |
| 13 | Q36 | .9144 | -.6261 |
| 14 | Q39 | .7258 | -1.5431 |

## Additional Cluster Analysis

The 14 survey items were examined using an ALSCAL three-dimensional analysis (see Table 4 and Figure 4). The three dimensions yielded an RSQ was .91301, meaning that any dimension would be represented. Therefore, the stress levels or measure of goodness of fit of the mapping solution to the data is .10913, a fair goodness-of-fit result.

Table 4

*Three-Dimension Analysis Results with Stimulus Coordinate Values*

| Stimulus Number | Stimulus Name | 1 | 2 | 3 |
|---|---|---|---|---|
| 1 | Q1 | .5344 | -.5411 | .0458 |
| 2 | Q5 | 1.9761 | .2311 | .8860 |
| 3 | Q6 | .2587 | -.2491 | .1825 |
| 4 | Q7 | 1.1396 | -1.0107 | .2417 |
| 5 | Q23 | .0629 | 1.8606 | .3016 |
| 6 | Q24 | -1.2730 | -.3411 | 1.5073 |
| 7 | Q26 | -.3757 | 2.4159 | .6473 |
| 8 | Q29 | -.9922 | -.9670 | .8889 |
| 9 | Q30 | -2.5212 | -.5685 | -1.1451 |
| 10 | Q31 | -.4663 | -.4178 | .4601 |
| 11 | Q32 | -.4646 | .3195 | -.8748 |
| 12 | Q33 | .3258 | .5065 | -1.0809 |
| 13 | Q36 | 1.1027 | -.7443 | -.1252 |
| 14 | Q39 | .6928 | -.4940 | -1.9352 |

## Additional Cluster Analysis

After considering the data analysis by the previously mentioned tests, it was determined that it was possible to eliminate several survey items. The following items were removed:

- Q5 VAL4 Mathematics is important in everyday life
- Q39 VAL10 A strong math background could help me in my professional life
- Q23 MOT1 I am confident that I could learn advanced mathematics
- Q30 ENJ7 I am happier in a mathematics class than in any other class
- Q26 ENJ4 I like to solve new problems in mathematics
- Q24 ENJ2 I have usually enjoyed studying mathematics in school

A shorter survey that was nearly balanced according to their original subfactors remained.

- Q7 VAL6 High school mathematics courses would be very helpful no matter what I decide to study
- Q6 VAL5 Mathematics is one of the most important subjects to study
- Q1 VAL1 Mathematics is a very worthwhile and necessary subject
- Q36 MOT5 I believe studying math helps me with problem-solving in other areas
- Q33 MOT4 I plan to take as much mathematics as I can during my education
- Q32 MOT3 I am willing to take more than the required amount of mathematics
- Q31 ENJ8 Mathematics is a very interesting subject
- Q29ENJ6 I really like mathematics

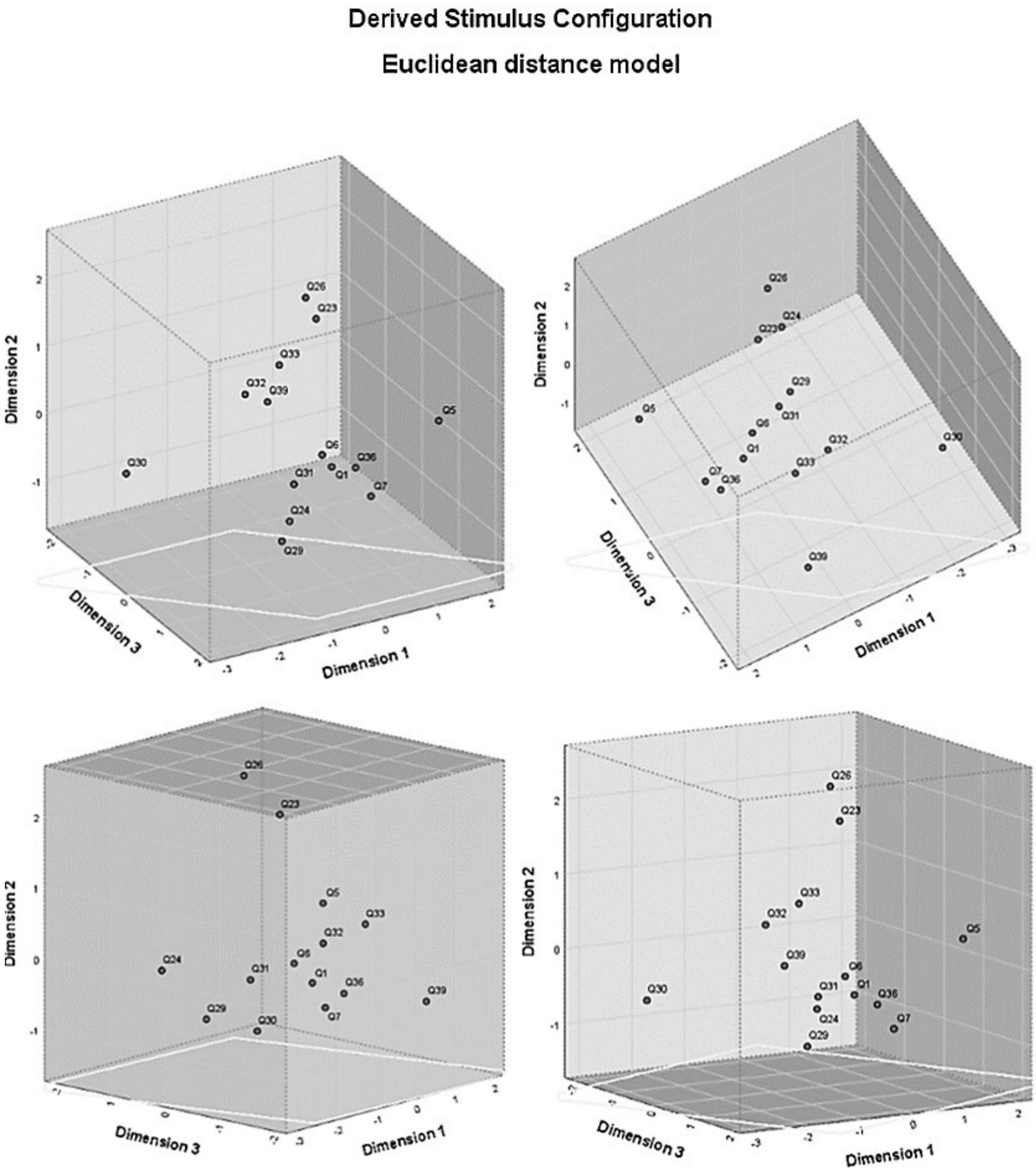


*Figure 4*. Survey Items Represented Three-Dimensionally – Four Different Views Shown

Running those two new factors against each other (the removed items against the remaining items) shows the cluster quality (figure 5) as "good" (see Table 6).

| | |
|---|---|
| **Algorithm** | TwoStep |
| **Inputs** | 2 |
| **Clusters** | 2 |

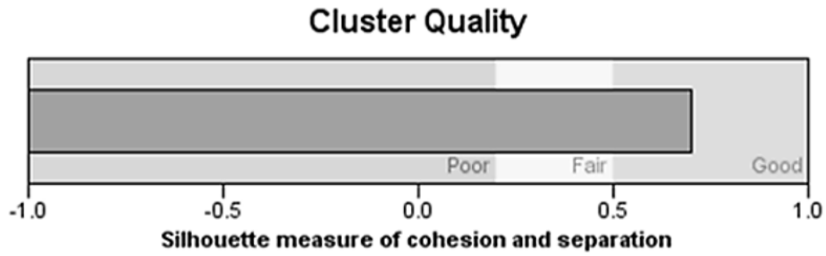


*Figure 5*.  Cluster Quality

Table 6

**Component Transformation Matrix**

| Component | 1 | 2 |
|---|---|---|
| 1 | .723 | .691 |
| 2 | .691 | -.723 |

Extraction Method: Principal Component Analysis.
Rotation Method: Varimax with Kaiser Normalization.

**Total Variance Explained**

| | Initial Eigenvalues | | | Extraction Sums of Squared Loadings | | |
|---|---|---|---|---|---|---|
| Component | Total | % of Variance | Cumulative % | Total | % of Variance | Cumulative % |
| 1 | 1.639 | 81.971 | 81.971 | 1.639 | 81.971 | 81.971 |
| 2 | .361 | 18.029 | 100.000 | | | |

Extraction Method: Principal Component Analysis.

## Conclusion

The purpose of this study was to examine if the Short Form ATMI was a valid instrument to use with upper elementary students.  One factor, self-confidence, showed poor internal reliability and was removed.  Two constructs were tested against race, revealing the internal reliability as "very good," which indicates that these selected items are measuring accurately with this group of participants.  The remaining 14-items were further analyzed.  When these items were examined, it revealed that an additional six items could be removed.  This deduction left the inventory with the remaining nine items, one under the desired goal of 10 or less.

Given its small size and short administration time, schools can administer this test to measure changes in attitudes over time.  Data collected will be analyzed with this larger sample size, and then its validity reexamined.

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