Lesson 2: Estimating Probabilities by Collecting Data

Student Outcomes

- Students estimate probabilities by collecting data on an outcome of a chance experiment.
- Students use given data to estimate probabilities.

Lesson Overview

This lesson builds on students' beginning understanding of probability. Lesson 1 introduced students to an informal idea of probability and the vocabulary *impossible*, *unlikely*, *equally likely*, *likely*, and *certain* to describe the chance of an event occurring. In this lesson, students begin by playing a game similar to the game they played in Lesson 1. The results of the game are used to introduce a method for finding an estimate for the probability of an event occurring. Then, students use data given in a table to estimate the probability of an event.

Classwork

Exercises 1–8 (18 minutes): Carnival Game

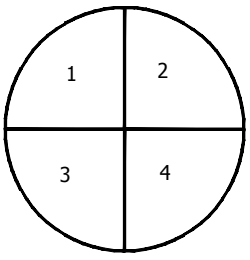
Place students into groups of two. Hand out a copy of the spinner and a paper clip to each group. Read through the rules of the game, and demonstrate how to use the paper clip as a spinner.

MP.2 Before playing the game, display the probability scale from Lesson 1, and ask students where they would place the probability of winning the game.

Remind students to carefully record the results of each spin.

Exercises 1–8: Carnival Game

At the school carnival, there is a game in which students spin a large spinner. The spinner has four equal sections numbered 1–4 as shown below. To play the game, a student spins the spinner twice and adds the two numbers that the spinner lands on. If the sum is greater than or equal to 5, the student wins a prize.



Students work with their partners on Exercises 1–8. Then, discuss and confirm as a class.

Sample responses to the questions should be based on the outcomes recorded by students. The following outcomes were generated by two students. They are used to provide sample responses to the questions that follow.

Play this game with your partner 15 times. Record the outcome of each spin in the table below.

Turn	1st Spin Results	2nd Spin Results	Sum
1	4	1	5
2	1	3	4
3	3	2	5
4	1	1	2
5	2	1	3
6	1	4	5
7	4	1	5
8	3	1	4
9	2	4	6
10	4	4	8
11	1	1	2
12	4	3	7
13	3	4	7
14	3	1	4
15	1	2	3

1. **Out of the 15 turns, how many times was the sum greater than or equal to 5?**

   Answers will vary and should reflect the results from students playing the game 15 times. In the example above, eight outcomes had a sum greater than or equal to 5.

2. **What sum occurred most often?**

   5 occurred the most.

3. **What sum occurred least often?**

   6 and 8 occurred the least. (Anticipate a range of answers, as this was only done 15 times. We anticipate that 2 and 8 will not occur as often.)

4. **If students were to play a lot of games, what fraction of the games would they win? Explain your answer.**

   Based on the above outcomes, $\frac{8}{15}$ represents the fraction of outcomes with a sum of 5 or more. To determine this, count how many games have a sum of 5 or more. There are 8 games out of the total 15 that have a sum of 5 or more.

5. **Name a sum that would be impossible to get while playing the game.**

   Answers will vary. One possibility is getting a sum of 100. Any sum less than 2 or greater than 8 would be correct.

6. **What event is certain to occur while playing the game?**

   Answers will vary. One possibility is getting a sum between 2 and 8 because all possible sums are between 2 and 8, inclusive.

Exercises 7–8 (5 minutes)

Before students work on Exercises 7 and 8, discuss the definition of a *chance experiment*. A *chance experiment* is the process of making an observation when the outcome is not certain (i.e., when there is more than one possible outcome). If students struggle with this idea, present some examples of a chance experiment such as flipping a coin 15 times or selecting a cube from a bag of 20 cubes. Then, display the formula for finding an estimate for the probability of an event. Using the game that students just played, explain that the denominator is the total number of times they played the game, and the numerator is the number of times they recorded a sum greater than or equal to 5.

> **When you were spinning the spinner and recording the outcomes, you were performing a *chance experiment*. You can use the results from a chance experiment to estimate the probability of an event. In Exercise 1, you spun the spinner 15 times and counted how many times the sum was greater than or equal to 5. An estimate for the probability of a sum greater than or equal to 5 is**
>
> $$P(\text{sum } \geq 5) = \frac{\text{Number of observed occurrences of the event}}{\text{Total number of observations}}.$$

Give students a few minutes to answer Exercises 7 and 8, and then ask each group to share their results. After students have shared their results, point out that not every group had exactly the same answer.

Ask students to explain why their answers are estimates of the probability of getting a sum of 5 or more.

MP.6

7. Based on your experiment of playing the game, what is your estimate for the probability of getting a sum of 5 or more?

Answers will vary. Students should answer this question based on their results. For the results indicated above, $\frac{8}{15}$ or approximately 0.53 or 53% would estimate the probability of getting a sum of 5 or more.

8. Based on your experiment of playing the game, what is your estimate for the probability of getting a sum of exactly 5?

Answers will vary. Students should answer this question based on their results. Using the above 15 outcomes, $\frac{4}{15}$ or approximately 0.27 or 27% of the time represents an estimate for the probability of getting a sum of exactly 5.

Students will learn how to determine a theoretical probability for problems similar to this game. Before they begin determining the theoretical probability, however, summarize how an estimated probability is based on the proportion of the number of specific outcomes to the total number of outcomes. Students may also begin to realize that the more outcomes they determine, the more confident they are that the proportion of winning the game is providing an accurate estimate of the probability. These ideas are developed more fully in the following lessons.

Example 2 (10 minutes): Animal Crackers

Students read the example. Consider showing a box of animal crackers and demonstrating how a student can take a sample from the box. Explain that the data presented resulted from a student taking a sample of 20 crackers from a very large jar of animal crackers and recording the results for each draw.

Display the table of data.

Ask students:

- What was the total number of observations?
  - *The total number of observations is* 20.
- How many zebras were chosen?
  - *Three zebras were chosen.*
- What is the estimate for the probability of selecting a zebra?
  - *The estimated probability for selecting a zebra is* $\frac{3}{20}$.

The main point of this example is for students to estimate the probability of selecting a certain type of animal cracker. Use the data collected to make this estimate.

Example 2: Animal Crackers

A student brought a very large jar of animal crackers to share with students in class. Rather than count and sort all the different types of crackers, the student randomly chose 20 crackers and found the following counts for the different types of animal crackers. Estimate the probability of selecting a zebra.

Animal	Number Selected
Lion	2
Camel	1
Monkey	4
Elephant	5
Zebra	3
Penguin	3
Tortoise	2
	Total 20

The estimated probability of picking a zebra is $\frac{3}{20}$, ***or*** 0.15 ***or*** 15%. ***This means that an estimate of the proportion of the time a zebra will be selected is*** 0.15 ***or*** 15% ***of the time. This could be written as*** $P(\text{zebra}) = 0.15$, ***or the probability of selecting a zebra is*** 0.15.

Exercises 9–15 (5 minutes)

Place students in groups of two to answer each question. Consider specifying in which form they should answer. For this exercise, it is acceptable for students to write answers in fraction form to emphasize the formula. As a class, briefly discuss students' answers. Specifically, discuss the answer for Exercise 11. This question involves "or." For this question, students should indicate that they would add the outcomes as indicated in the question to form their proportions.

Exercises 9–15

If a student randomly selected a cracker from a large jar:

9. **What is your estimate for the probability of selecting a lion?**

   $\frac{2}{20} = \frac{1}{10} = 0.1$

10. **What is your estimate for the probability of selecting a monkey?**

    $\frac{4}{20} = \frac{1}{5} = 0.2$

11. **What is your estimate for the probability of selecting a penguin or a camel?**

    $\frac{(3+1)}{20} = \frac{4}{20} = \frac{1}{5} = 0.2$

12. **What is your estimate for the probability of selecting a rabbit?**

    $\frac{0}{20} = 0$

13. **Is there the same number of each kind of animal cracker in the jar? Explain your answer.**

    No. There appears to be more elephants than other types of crackers.

14. **If the student randomly selected another 20 animal crackers, would the same results occur? Why or why not?**

    Probably not. Results may be similar, but it is very unlikely they would be exactly the same.

15. **If there are 500 animal crackers in the jar, approximately how many elephants are in the jar? Explain your answer.**

    $\frac{5}{20} = \frac{1}{4} = 0.25$; *hence, an estimate for the number of elephants would be* 125 *because* 25% *of* 500 *is* 125.

Closing (2 minutes)

- What information do you need to determine the estimated probability?
  - *I need to know the total number of observations and the number of observed occurrences of a specific event.*

Discuss the Lesson Summary with students. Ask students to summarize how they would find the probability of an event.

Lesson Summary

An estimate for finding the probability of an event occurring is

$$P(\text{event occurring}) = \frac{\text{Number of observed occurrences of the event}}{\text{Total number of observations}}$$

Exit Ticket (5 minutes)

Name _____ Date_____

Lesson 2: Estimating Probabilities by Collecting Data

Exit Ticket

In the following problems, round all of your decimal answers to three decimal places. Round all of your percents to the nearest tenth of a percent.

A student randomly selected crayons from a large bag of crayons. The table below shows the number of each color crayon in a bag. Now, suppose the student were to randomly select one crayon from the bag.

Color	Number
Brown	10
Blue	5
Yellow	3
Green	3
Orange	3
Red	6

1. What is the estimate for the probability of selecting a blue crayon from the bag? Express your answer as a fraction, decimal, or percent.

2. What is the estimate for the probability of selecting a brown crayon from the bag?

3. What is the estimate for the probability of selecting a red crayon *or* a yellow crayon from the bag?

4. What is the estimate for the probability of selecting a pink crayon from the bag?

5. Which color is most likely to be selected?

6. If there are 300 crayons in the bag, how many red crayons would you estimate are in the bag? Justify your answer.

Exit Ticket Sample Solutions

In the following problems, round all of your decimal answers to three decimal places. Round all of your percents to the nearest tenth of a percent.

A student randomly selected crayons from a large bag of crayons. The table below shows the number of each color crayon in a bag. Now, suppose the student were to randomly select one crayon from the bag.

Color	Number
Brown	10
Blue	5
Yellow	3
Green	3
Orange	3
Red	6

1. **What is the estimate for the probability of selecting a blue crayon from the bag? Express your answer as a fraction, decimal, or percent.**

   $\frac{5}{30} = \frac{1}{6} \approx 0.167$ *or* 16.7%

2. **What is the estimate for the probability of selecting a brown crayon from the bag?**

   $\frac{10}{30} = \frac{1}{3} \approx 0.333$ *or* 33.3%

3. **What is the estimate for the probability of selecting a red crayon *or* a yellow crayon from the bag?**

   $\frac{9}{30} = \frac{3}{10} = 0.3 = 30\%$

4. **What is the estimate for the probability of selecting a pink crayon from the bag?**

   $\frac{0}{30} = 0\%$

5. **Which color is most likely to be selected?**

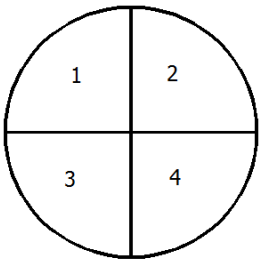
   Brown

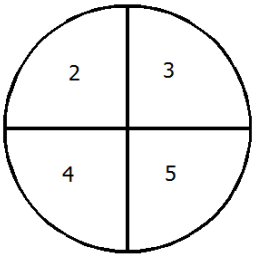
6. **If there are 300 crayons in the bag, how many red crayons would you estimate are in the bag? Justify your answer.**

   There are 6 out of 30, or $\frac{1}{5}$ or 0.2, crayons that are red. Anticipate $\frac{1}{5}$ of 300 crayons are red, or approximately 60 crayons.

Problem Set Sample Solutions

1. Play a game using the two spinners below. Spin each spinner once, and then multiply the outcomes together. If the result is less than or equal to 8, you win the game. Play the game 15 times, and record your results in the table below. Then, answer the questions that follow.





Turn	1st Spin Results	2nd Spin Results	Product
1	2	2	4
2	1	2	4
3	4	3	12
4	3	3	9
5	1	4	4
6	4	5	20
7	3	2	6
8	4	4	16
9	2	5	10
10	1	2	2
11	3	2	6
12	1	3	3
13	3	4	12
14	2	4	8
15	1	5	5

a. What is your estimate for the probability of getting a product of 8 or less?

Answers should be approximately 7, 8, or 9 divided by 15. The probability for the sample spins provided is $\frac{9}{15}$, *or* $\frac{3}{5}$.

b. What is your estimate for the probability of getting a product of more than 8?

Subtract the answer to part (a) from 1, or 1 − the answer from part (a). Approximately 8, 7, or 6 divided by 15. The probability for the sample spins provided is $\frac{6}{15}$, *or* $\frac{2}{5}$.

c. What is your estimate for the probability of getting a product of exactly 8?

Approximately 1 or 2 divided by 15. The probability for the sample spins provided is $\frac{1}{15}$.

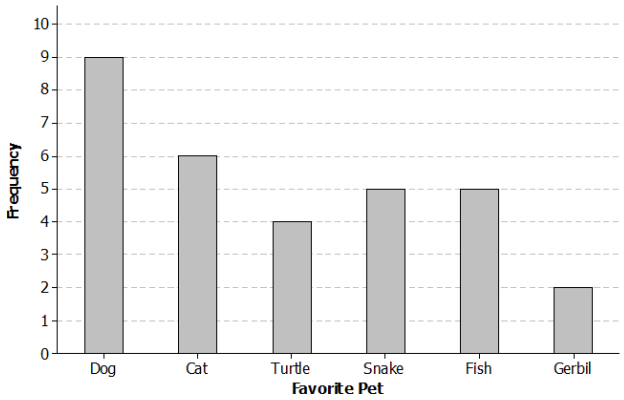
d. What is the most likely product for this game?

Possibilities are 4, 6, 8, and 12. The most likely product in the sample spins provided is 4.

e. If you play this game another 15 times, will you get the exact same results? Explain.

No. Since this is a chance experiment, results could change for each time the game is played.

2. A seventh-grade student surveyed students at her school. She asked them to name their favorite pets. Below is a bar graph showing the results of the survey.



Use the results from the survey to answer the following questions.

a. How many students answered the survey question?

31

b. How many students said that a snake was their favorite pet?

5

Now, suppose a student is randomly selected and asked what his favorite pet is.

c. What is your estimate for the probability of that student saying that a dog is his favorite pet?

(Allow any form.) $\frac{9}{31}$, *or approximately* 0.29 *or approximately* 29%

d. What is your estimate for the probability of that student saying that a gerbil is his favorite pet?

(Allow any form.) $\frac{2}{31}$, *or approximately* 0.06 *or approximately* 6%

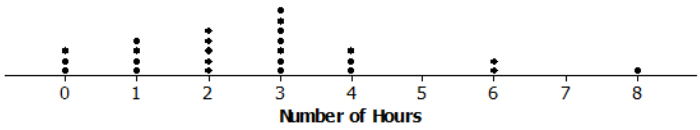
e. What is your estimate for the probability of that student saying that a frog is his favorite pet?

$\frac{0}{31}$, *or* 0 *or* 0%

3. A seventh-grade student surveyed 25 students at her school. She asked them how many hours a week they spend playing a sport or game outdoors. The results are listed in the table below.

Number of Hours	Tally	Frequency
0	| | |	3
1	| | | |	4
2	𝍸	5
3	𝍸 | |	7
4	| | |	3
5		0
6	| |	2
7		0
8	|	1

a. Draw a dot plot of the results.



Suppose a student will be randomly selected.

b. What is your estimate for the probability of that student answering 3 hours?

$\frac{7}{25} = 0.28 = 28\%$

c. What is your estimate for the probability of that student answering 8 hours?

$\frac{1}{25} = 0.04 = 4\%$

d. What is your estimate for the probability of that student answering 6 or more hours?

$\frac{3}{25} = 0.12 = 12\%$

e. What is your estimate for the probability of that student answering 3 or fewer hours?

$\frac{19}{25} = 0.76 = 76\%$

f. If another 25 students were surveyed, do you think they would give the exact same results? Explain your answer.

No. Each group of 25 students could answer the question differently.

g. If there are 200 students at the school, what is your estimate for the number of students who would say they play a sport or game outdoors 3 hours per week? Explain your answer.

$200 \cdot \left(\frac{7}{25}\right) = 56$

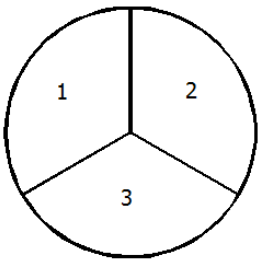
I would estimate that 56 students would say they play a sport or game outdoors 3 hours per week. This is based on estimating that, of the 200 students, $\frac{7}{25}$ would play a sport or game outdoors 3 hours per week, as $\frac{7}{25}$ represented the probability of playing a sport or game outdoors 3 hours per week from the seventh-grade class surveyed.

4. A student played a game using one of the spinners below. The table shows the results of 15 spins. Which spinner did the student use? Give a reason for your answer.

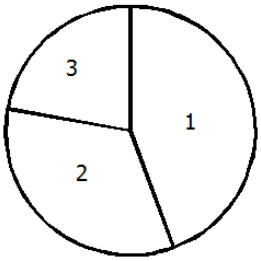
Spinner B. Tallying the results: 1 occurred 6 times, 2 occurred 6 times, and 3 occurred 3 times. In Spinner B, the sections labeled 1 and 2 are equal and larger than section 3.

Spin	Results
1	1
2	1
3	2
4	3
5	1
6	2
7	3
8	2
9	2
10	1
11	2
12	2
13	1
14	3
15	1

Spinner A



Spinner B



Spinner C